Perfect Price Discrimination with Costless Arbitrage*

by

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The ability of a monopoly seller to prevent resale is often presented as a necessary condition for first degree price discrimination. In this paper, we explore this claim and show that, even with costless arbitrage markets, price discrimination may continue to be both feasible and profit maximising despite potential resale. With finite numbers of consumers, arbitrage markets may be 'thin', in the sense that there can be too few low-valuation consumers to supply high-valuation consumers. We examine both *ex ante* and *ex post* arbitrage markets and show how a monopoly can exploit potential 'thinness' to profitably price discriminate. In each case, we present sufficient conditions for equilibrium price discrimination. We note that the form of such discrimination depends on the nature of the arbitrage market and consider business strategies that a monopoly might adopt to exacerbate market thinness. Our results show how market depth and the effectiveness of arbitrage can be the key elements for price discrimination, rather than the per se prevention of reselling. *Journal of Economic Literature* Classification Numbers: D42, L11

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1. Introduction

According to most treatments (e.g., Varian, 1989), price discrimination requires firms to (1) have some market power, (2) be able to sort consumers and (3) be able to prevent resale. When it comes to the benchmark case of first degree or perfect price discrimination the first two conditions become stronger in that the firm must also be able to make take it or leave it offers to consumers and possess perfect information regarding a consumer's type.

This paper explores the possibility of first degree price discrimination in an environment where (3), the no resale condition, is not satisfied. It is commonly asserted that this change to the standard treatment undermines the possibility of price discrimination. Consider, for instance:

> It is clear that if the transaction (arbitrage) costs between two consumers are low, any attempt to sell a given good to two consumers at different prices runs into the problem that the low-price consumer buys the good to resell it to the high-price one. (Tirole, 1988, p.134)

To our knowledge there have been no attempts to model the resale market to test whether it is in fact true that this possibility undermines price discrimination.[1]

To explore this, we completely relax (3) and assume the opposite: that resale can occur free of transaction or transportation costs. Moreover, we assume that any trader – the monopolist producer or arbitrageurs – can make take it or leave it offers to any consumer and knows that consumer's type. Despite the complete relaxation of the no resale condition, we demonstrate that price discrimination is still both feasible and potentially profitable for a monopolist seller.

Our contribution highlights the importance of the number of low valuation consumers in driving price discrimination. In a model with two consumer types and a monopolist with unlimited capacity, we model two variants of resale. In the first, following an initial round of sales by the monopolist, the monopolist and initial purchasers can act as sellers in an ex post market. We term this *ex post arbitrage*. In this

[1] An exception is Alger (1999) who demonstrates that arbitrage, in a model where consumer types are private information, can eliminate price discrimination.

situation, we demonstrate that there are conditions under which the monopolist sells to all consumers initially and charges high-valuation types a higher price than the price to low-valuation types. The possibility of ex post arbitrage constrains the monopolist's pricing to high-valuation consumers but does not prevent them from engaging in price discrimination. The reason is that, in equilibrium, the high-valuation consumers are not certain that low-valuation consumers will trade in the ex post market and are, therefore, willing to accept a higher up-front price. There is no discounting or risk aversion driving this result although the presence of either would strengthen it.

In the second variant, we consider forward markets for the sale of the monopolist's product. In these markets, low-valuation consumers can sell forward contracts to high-valuation ones and settle these by purchasing the requisite stock from the monopolist later on. In this situation, it may be worthwhile for the monopolist to set a pricing policy equivalent to a perfect price discrimination outcome whereby each consumer is charged their willingness to pay. This is because the monopolist has an incentive to speculate on there being insufficient low types for trade to have occurred in the forward market rather than foreclosing on that possibility by setting a single high price.

In each case, it is *uncertainty regarding the resale market* – specifically, the possibility that a re-sale market may be 'thin' – that drives the result. Put simply, when a monopolist can observe a consumer's type, the low-valuation consumers are those who can perform an arbitrage as well as a consumption function. The monopolist can choose to foreclose on these by not selling to those consumers (i.e., charging a high price) or, if there is a strong enough possibility that the numbers of such consumers may be low, sell to them and price discriminate. As such, the main contribution of this paper is to identify a new condition that permits price discrimination to arise – the expected number of low type consumers – and, in so doing, improve our understanding of the foundations of price discrimination. In addition, while our model is extreme in that a monopolist can perfectly observe a consumer's type and competition in the re-sale market is strong, it identifies strategies that enhance expectations that re-sale markets will be thin, such as volume purchase constraints, which may accompany practices of price discrimination.

The paper proceeds as follows. Section 2 outlines the basic model structure and the textbook case of perfect price discrimination; to serve as a benchmark. Section 3 considers the case of ex post arbitrage while Section 4 analyses what happens if there is an ex ante or forward market. A final section concludes.

2. The Model and Perfect Price Discrimination

We consider a simple setting where there is a finite set of n consumers in a market who each value one unit (and only one unit) of a product manufactured by a single producer, M. The producer's marginal cost of production is a constant c per unit. M is not capacity constrained in any time period.

There are two 'types' of consumers who differ according to their willingness-to-pay for M's product: L types (L) who value the product at $\underline{v}$ and H types (H) who value it at $\overline{v}$, where $\underline{v} < \overline{v}$. We assume that $\underline{v} > c$. The probability that any given consumer has a low valuation is π and this probability is independent across consumers. To avoid trivial outcomes, we assume that π is strictly between zero and one. Finally, the model below involves two periods and we assume that there is no discounting.[2]

A standard approach to perfect price discrimination shows how the profit-maximizing prices set by M differ according to the information available to M. Suppose that M has no information about any consumer's type, consumers have no interaction with each other either before or after dealing with M and that M must set prices prior to any consumer purchasing its product. In this situation, M will post a single price, p, for all consumers. There are two alternative profit-maximizing prices that M could set. First, M could set $p = \underline{v}$. Its expected profit per consumer will be $\underline{v} - c$ and it makes sales to all n consumers. Alternatively, M could set $p = \overline{v}$. Only H-types purchase the product and M's expected profit per consumer is $(1-\pi)(\overline{v} - c)$.[3] M will choose the price that maximises the expected profit per customer.

[2] This avoids trivial cases where price discrimination might arise because some consumers prefer to buy off a monopolist at a high price today rather than wait for a lower price in the future. In other words, this assumption removes one potential source of price discrimination.

[3] It is straightforward to show that other possible prices are not profit maximising for M.

In contrast, suppose that consumers have no direct interaction with each other and that *M* must set prices prior to any consumer purchasing its product, but allow *M* to observe a consumer's type prior to sale. That is, the minute a consumer walks into a store their type is revealed to *M* and *M* can make that customer a take it or leave it offer. Then it is profit maximising for *M* to set two prices; a price to *L* types of $\underline{p} = \underline{v}$ and to *H* types of $\overline{p} = \overline{v}$ giving it expected profits per consumer of $(1-\pi)\overline{v} + \pi\underline{v} - c$. This is the textbook outcome from *perfect price discrimination*.

In the analysis below we retain the assumptions required for perfect price discrimination – monopoly power and consumer type observability – except that we will allow consumers to interact with each other either after purchasing from *M* or before purchasing from *M*. *M* will be able to make each consumer a take it or leave it offer at the price set for that consumer's type. We denote the take it or leave it price offers as $\overline{p}$ and $\underline{p}$ to *H* and *L* types respectively. As with the example of perfect price discrimination given above, before setting its prices *M* does not know for sure how many consumers of a given type there are in the market. Similarly, while consumers know their own valuation they do not know the valuations of other consumers and, thus, do not know for sure the number of *H* and *L* consumers in the population.

Unlike the example of perfect price discrimination, however, consumers will be able to meet and to trade with each other either prior to purchasing from *M* or after purchasing from *M*. If consumers meet after purchasing from *M* then they may trade the relevant product. We refer to this as *ex post* arbitrage. If consumers meet before purchasing from *M* then they will be able to trade 'futures contracts' for delivery of the relevant product. We refer to this as *ex ante* arbitrage.

3. Ex Post Arbitrage

First, consider the situation where consumers are able to interact and trade after the initial round of sales from *M*. The timing for this is as follows:

PERIOD 1: *M* makes (unit) price offers of $\overline{p}$ and $\underline{p}$ to *H* and *L* types respectively and consumers choose their quantities.

PERIOD 2: *M* and purchasers in period 1 compete on the basis of price to sell to any consumers who chose not to purchase in period 1. All consumers holding the product at the end of the period consume it.

Thus, *M* makes an initial round of sales to prospective consumers. Some of those may purchase while others may choose to wait until the *ex post* market. In that market, *M* and purchasers in the initial round both have the opportunity to sell to any remaining consumers.

Our aim here is to consider whether price discrimination remains viable in period 1 despite the existence of a well-functioning *ex post* arbitrage market. Price discrimination will arise in period 1 if it is profit maximizing for *M* to set prices $\overline{p} > \underline{p} \geq \underline{v}$ with at least some of each type of consumer buying at their relevant price in the first period. Clearly, behaviour in the first period will depend on expectations about the arbitrage market. As such, the form of competition and price formation in the period 2 market is critical.

To make things as stark as possible, we adopt here an extreme model of competition in period 2. Specifically, we assume that:

1. **Price discrimination**: in the period 2 market, in the absence of any other sellers, *M* is able to perfectly price discriminate.
2. **Intense price competition**: if there exists *any consumer* who has been sold the good in period 1, then the realised period 2 price will equal the lowest opportunity cost of consuming the good for those consumers have purchased in period 1.

The first assumption is a natural one in that *M*, as a monopolist, faces the same ability to distinguish between customer types in period 2 as it does in period 1.

The second assumption is a strong one as it means that, should *M* choose to sell to any *L*-types in period 1, it will be constrained in its pricing in period 2. Specifically, that *L*-type will have a valuation of the good in consumption of $\underline{v}$. If it only has one unit, for a price greater than $\underline{v}$, an *L*-type will be willing to sell the good in period 2. On the other hand, if an *L*-type has purchased multiple units, its opportunity cost of period 2 sales will be 0. Thus, we do not rule out the possibility that *L*-types could purchase more than a single unit in the first period, so that *M* can face potentially strong price competition in period 2. We will comment below on what happens when this second assumption is relaxed.

There are some clear implications of these assumptions on the operation of the period 2 market:

- If there are no *L* consumers then the period 2 price will be $\overline{v}$. This follows as, by assumption 2, the realized period 2 price can only fall below $\overline{v}$ if an *H* type bought multiple units in period 1. But purchasing multiple units can never be expected to be strictly profitable for one *H* type as it would involve selling to another *H* type in period 2 who would then expect to be better off purchasing in period 1.
- If there are no *H* consumers in period 2 or if there is at least one *L* consumer and all *H* consumers purchase in period 1 then the period 2 price is no greater than $\underline{v}$. This is a direct implication of the fact that no *L* consumer will pay more than $\underline{v}$ in period 2.
- The realised period 2 price can only be strictly less than $\underline{v}$ if at least one *L* consumer makes multiple purchases in period 1. If an *L* consumer has multiple units, their opportunity cost of selling the marginal unit is 0. Otherwise, for those consumers, their opportunity cost is $\underline{v}$ so *M* can set a period 2 price of $\underline{v}$ and still make sales.

Given our assumptions, it may seem that price discrimination in period 1 is not profitable. After all, if *any* *L*-type consumer purchases in the first period, the second period price cannot exceed $\underline{v}$. This leads to a natural question as to why an *H*-type consumer purchases in the first period at a price above $\underline{v}$. However, we show below that price discrimination is still possible because, from the perspective of an *H* consumer in period 1, the expected second period price is greater than $\underline{v}$ even under our intense competition assumptions. Indeed, we demonstrate below that the chief implication of our model is that the expected period 2 price will be higher the lower is the probability that there are *L*-type consumers present in the market. This probability is, of course, partially controlled by *M* in its period 1 pricing choices.

Immediately after *M* sets period 1 prices but prior to any purchases, both *M* and consumers will have expectations of the second period price. These expectations, however, will depend on each agent's identity. When *M* sets the period 1 prices it has no information about the specific types of any consumers. In contrast, as consumers know their own type, they have some information that is relevant to the second period price. Specifically, an *H*-type expects that the period 2 price will be higher than *M*'s expectation as it knows the probability that there is at least one *L*-type they might trade with is lower.

To see this, suppose that each agents' forecast of the expected period 2 equilibrium price when there are *k* *L*-types is $p_r(k)$. The probabilities, $\Pi(k;i)$, that each agent, *i*, assigns to there being *k* *L*-types differs because of their prior information. For *M*, it is $\Pi(k;M) = C_k^n (1-\pi)^{n-k}\pi^k$ while for *L* and *H* it is $\Pi(k;L) = C_{k-1}^{n-1}(1-\pi)^{n-k}\pi^{k-1}$ and $\Pi(k;H) = C_k^{n-1}(1-\pi)^{n-k-1}\pi^k$, respectively. Thus, agent *i*'s expected period 2 price, $E[p_r;i]$, is given by:

Then:

$$E[p_r;M] = \Pi(0;M)\,p_r(0) + \ldots + \Pi(n;M)\,p_r(n) \quad (1)$$

$$E[p_r;L] = \Pi(1;L)\,p_r(1) + \ldots + \Pi(n;L)\,p_r(n) \quad (2)$$

$$E[p_r;H] = \Pi(0;H)\,p_r(0) + \ldots + \Pi(n-1;H)\,p_r(n-1) \quad (3)$$

Notice that if $p_r(k)$ is non-increasing in *k*, $E[p_r;L] \le E[p_r;M] \le E[p_r;H]$. Notice also that if *M* has set period 1 prices $\overline{p} > \underline{p} \ge \underline{v}$ then prior to any consumer purchasing, $E[p_r;L] \ge \underline{v}$. If not, this would imply the some *L*-types have purchased units in period 1 with the expectation of selling them for less in period 2. Those *L*-types would be better off by not making such purchases.

Given this, we can demonstrate the following in relation to the profitability of price discrimination:

Proposition 1: *The producer M will always price discriminate in period 1 if*

$$\pi(\underline{v}-c) > (1-\pi)\left(1-(1-\pi)^{n-1}\right)(\overline{v}-\underline{v}) .$$

PROOF: To demonstrate this, we will conjecture equilibrium choices of period 1 prices by *M* and demonstrate that this is the unique equilibrium outcome. As such, suppose that $\overline{p} = v^*$ and $\underline{p} = \underline{v}$ where $v^* = \Pi(0;H)\overline{v} + \left(1 - \Pi(0;H)\right)\underline{v}$ and $\Pi(0;H) = (1-\pi)^{n-1}$. Given this,

$$p_r(k) = \begin{cases} \overline{v} & \text{if } k = 0 \\ \underline{v} & \phantom{\text{if }} k > 0 \end{cases}$$

Thus, $v^* = E[p_r;H]$ and it is strictly greater than $\underline{v}$ if $\Pi(0;H) > 0$. Thus, at this price, an *H*-type will be indifferent between purchasing in period 1 and waiting until period 2 so will, by assumption, purchase in period 1. In this case, *M*'s expected profits per customer are: $\pi\underline{v} + (1-\pi)v^* - c$ as all consumers purchase in period 1. Notice also that $E[p_r;L] = \underline{v}$ and all *L* consumers will purchase one and only one unit in period 1. To purchase more will lead to $E[p_r;L] < \underline{v}$ which will be unprofitable.

Will *M* set $\underline{p} \in [c, \underline{v})$? Notice that, in so doing, this does not change expected prices in period 2 as no *L*-type chooses to purchase an extra unit at this price with the intention of selling in period 2 as it would be under-cut by *M*. Thus, this just reduces M's expected profits by $\pi(\underline{v} - \underline{p})$. Similarly, by setting $\underline{p} > \underline{v}$, *L*-types would not purchase the good at all as they would continue to expect under-cutting from *M* in period 2. However, this would cause $E[p_r;H] = \overline{v}$. Thus, *M* would only find this profitable if it found setting a single price of $\overline{v}$ profitable. That is, if $(1-\pi)(\overline{v} - c) \geq \pi\underline{v} + (1-\pi)v^* - c$. This will not occur if this inequality did not hold as is stated in the condition of the proposition.

Finally, given that it sets $\underline{p} = \underline{v}$ will *M* want to set $\overline{p}$ other than at $\overline{v}$. A higher level would leave it with only *L*-types purchasing and a strictly lower profit. A lower level would leave it with the same volume of sales but at a lower margin; also reducing profits. Hence, it is profit maximising for *M* to set prices according to the conjectured equilibrium.

Thus, if either the probability of any individual consumer being *L* is high enough and/or if $\overline{v}$ is close enough to $\underline{v}$ then *M* will always price discriminate in period 1. This makes intuitive sense. In the face of *ex post* arbitrage, price discrimination allows *M* to gain profits from *L* consumers as well as *H* consumers. However, the existence of the arbitrage market limits the amount of surplus that *M* can extract from the high valuation consumers. Thus, price discrimination is only optimal for *M* if the gain in profit from low valuation consumers offsets the loss in profit from *H* consumers.

The reason why the period 2 arbitrage market does not eliminate price discrimination is evident from an *H*-types' expected period 2 price in equilibrium:

$v^* = (1-\pi)^{n-1}\overline{v} + \left(1-(1-\pi)^{n-1}\right)\underline{v}$. Note that as $(1-\pi)^{n-1}$ approaches zero (for example as *n* becomes large), v^* approaches $\underline{v}$ and the scope for profitable price discrimination falls. Thus, the ability of *M* to price discriminate critically depends on the likelihood of the event that there will be no *L* consumers in the second period so that any *H* consumer who fails to buy in the first period will face the risk of a high price in the second period. It is the risk that there are no *L* consumers that undermines the effectiveness of *ex post* arbitrage.

It is useful to note here that the profitability of price discrimination is related to the intensity of price competition in the period 2 market. Specifically, softer price competition would lead to expected period 2 prices being above $\underline{v}$ for more states than when there are simply no *L*-types at all. For instance, period 2 competition could be Bertrand with capacity constraints which sometimes results in a mixed strategy equilibrium with prices above opportunity cost. Given uncertainty regarding the precise number of *L*-types, this will increase the $\overline{p}$ that *M* can charge in period 1 to prevent *H*-types from waiting until the period 2 market. In this respect, Proposition 1 could be viewed as presenting sufficient conditions so that *M* will always choose to price discriminate in period 1 even when period 2 price competition takes a more general form.

4. Ex Ante Arbitrage

As noted earlier, arbitrage can also occur via forward markets – something we term *ex ante* arbitrage. In this situation, consumers trade forward purchase agreements with other consumers. In so doing, consumers have knowledge of the type of consumer they are trading with. A consumer who sells a forward contract is then obligated to purchase the requisite goods for that contract from *M* and they will be charged a price based on their type and not on the type of consumer they are selling to.

The timing of the *ex ante* forward market is as follows:

PERIOD 0: *L* and *H* consumers trade forward purchase contracts.

PERIOD 1: *M* makes (unit) price offers of $\overline{p}$ and $\underline{p}$ to *H* and *L* types respectively and low and high types choose their quantities. Forward contracts are settled and consumers holding the product consume it.

To analyze this, we need to place some minimal constraints on the behaviour in the forward market and on the interaction between the forward market and the period 1 ‘spot’ market.

In the forward market, there are clearly a wide variety of forward contracts that could potentially be traded. We limit attention to a highly-flexible subset of these contracts that are relevant for price discrimination: a forward contract will involve the seller agreeing to deliver the purchaser one unit of the relevant product at the end of period 1, contingent on the period 1 price $\underline{p}$ not exceeding a particular value. Thus, we can denote a forward contract by its critical price *f*. If a consumer holds one forward contract with critical price *f* in period 1 then, if $\underline{p} \leq f$, that consumer will receive one unit of the product from the counterparty in period 1. If, however, $\underline{p} > f$, then the holder of the forward contract receives nothing. To eliminate some complexity, we also assume that all forward contracts are rendered void if $\bar{p} < \underline{p}$ and that only *L* consumers sell forward contracts and these are only purchased by *H* consumers. This latter condition (which holds automatically in the equilibrium below) allows us to focus on price discrimination and avoids trivial issues such as forward trade between consumers with identical valuations. Importantly, this also implies that if there is only one type of consumer then there are no trades in the forward market.

An immediate implication of these assumptions is that there is no trade in forward contracts with critical price $f > \bar{v}$. This reflects the fact that there will be no trade in contracts which might be exercised even though the relevant period 1 price exceeds the value of the product to an *H* consumer.

The forward markets and spot markets will be linked. When *M* sets the prices for its product, these prices may depend on whether or not *M* can observe the outcome of the forward market and, if *M* can observe this, what information if any the forward market conveys to *M*. We begin by making the simplest assumption, that *M* cannot observe the price or trades in the forward market. However, it will still have beliefs over that outcome. Let $P(f)$ denote the probability that *M* places on an *H* consumer having purchased a forward contract with a critical price no lower than *f*. It is reasonable to assume that $P(f)$ is non-increasing in *f*.

Given that *M* learns no relevant information from the forward market then, regardless of the forward trades that actually occur, *M* will set prices according to the following lemma.

Lemma 1: *Without loss of generality, we can restrict attention to situations where M sets profit maximising prices* $\underline{p} \in \{\underline{v}, \overline{v}\}$ *and* $\overline{p} = \overline{v}$ *in period 1.*

PROOF: In period 1, *M* has a number of mutually exclusive choices:

(i) *M* could set $\underline{p} \le \overline{p} \le \underline{v}$ and gain expected per consumer profits of $\pi(\underline{p}-c)+(1-\pi)P(\underline{p})(\underline{p}-c)+(1-\pi)(1-P(\underline{p}))(\overline{p}-c)$;

(ii) *M* could set $\underline{p} \le \underline{v} < \overline{p} \le \overline{v}$ and gain expected per consumer profits of $\pi(\underline{p}-c)+(1-\pi)P(\underline{p})(\underline{p}-c)+(1-\pi)(1-P(\underline{p}))(\overline{p}-c)$;

(iii) *M* could set $\underline{v} < \underline{p} \le \overline{p} \le \overline{v}$ and gain expected per consumer profits of $(1-\pi)P(\underline{p})(\underline{p}-c)+(1-\pi)(1-P(\underline{p}))(\overline{p}-c)$;

(iv) *M* could set $\underline{v} < \underline{p} \le \overline{v} < \overline{p}$ and gain expected per consumer profits of $(1-\pi)P(\underline{p})(\underline{p}-c)$;

(v) *M* could set $\overline{v} < \underline{p} \le \overline{p}$ and make zero profits;

(vi) *M* could set $\underline{p} > \overline{p}$.

Further, (i) to (vi) fully characterize *M*'s pricing options in period 1.

Profits in (i), (ii) and (iii) are increasing in $\overline{p}$. So in (i) *M* will set $\overline{p} = \underline{v}$ while in (ii) and (iii) *M* will set $\overline{p} = \overline{v}$. Given this, and remembering that $P(f)$ is non-increasing in *f*, it is easy to see in (i) that *M*'s profit maximizing prices are $\underline{p} = \overline{p} = \underline{v}$ which gives *M* per customer profits of $\underline{v}-c$. Similarly, in (ii), *M*'s profit maximizing prices are $\underline{p} = \underline{v}$ and $\overline{p} = \overline{v}$ yielding expected per customer profit $[\pi+(1-\pi)P(\underline{v})](\underline{v}-c)+(1-\pi)(1-P(\underline{v}))(\overline{v}-c)$. In (iii), *M*'s profit maximizing prices are $\underline{p} = \overline{p} = \overline{v}$ with profit $(1-\pi)(\overline{v}-c)$.

Note that in each of these cases, profits are strictly positive so that any of these cases is preferable to (v). Further, it is easy to confirm that (iii) yields at least as high profits as (iv) and that (ii) yields at least as high profits as (i). Thus, if *M* sets $\underline{p} \le \overline{p}$, it will be profit maximizing to set $\underline{p} \in \{\underline{v}, \overline{v}\}$ and $\overline{p} = \overline{v}$.

Finally, suppose that *M* sets $\underline{p} > \overline{p}$. It is easy to show that this leads to profits for *M* strictly less than $\underline{v}-c$ if $\underline{p} \le \underline{v}$. But this is less than the profits from (i) so this can never be profit maximizing. If $\underline{p} > \underline{v}$ then *M*'s profits are $(1-\pi)(\overline{p}-c)$ if $\overline{p} \le \overline{v}$ and zero otherwise. Thus if *M* sets $\underline{p} > \underline{v}$ then its profit maximizing prices are $\overline{v} = \overline{p} < \underline{p}$ and this yields the same profit as the maximized profits in (iii). Thus without loss of generality we can restrict attention to situations where *M* sets profit maximizing prices $\underline{p} \in \{\underline{v}, \overline{v}\}$ and $\overline{p} = \overline{v}$.

From Lemma 1, we can reduce the profit maximising choice for *M* to two alternatives. If *M* price discriminates and chooses $\underline{p} = \underline{v}$ and $\overline{p} = \overline{v}$ then its expected per customer profit is $[\pi + (1-\pi)P(\underline{v})](\underline{v} - c) + (1-\pi)(1-P(\underline{v}))(\overline{v} - c)$. Alternatively, if *M* does not price discriminate and chooses $\underline{p} = \overline{p} = \overline{v}$ then its expected per customer profit is $(1-\pi)(\overline{v} - c)$. Comparing these two options, *M* will always price discriminate if $\pi(\underline{v} - c) - (1-\pi)P(\underline{v})(\overline{v} - \underline{v}) > 0$. The presence of a forward market reduces *M*'s incentive to price discriminate as $P(\underline{v}) > 0$ because setting discriminatory prices will lead to *H* consumers exercising their forward contracts and effectively buying the product at the *L* price.

Clearly, the likelihood of price discrimination depends on the exact functioning of the forward market and, in general, this requires a specific market model. However, it is possible to bound $P(\underline{v})$ given our restrictions on the forward market. In particular, we know that $P(\underline{v}) \leq 1 - (1-\pi)^n$. This follows because, if there are no *L* consumers, then no *H* consumer can purchase a forward contract. Thus:

Proposition 2: *The producer M will always price discriminate in period 1 if* $\pi(\underline{v} - c) - (1-\pi)\left(1 - (1-\pi)^n\right)(\overline{v} - \underline{v}) > 0$.

At this stage, it is worth comparing Propositions 1 and 2. Note that, as in the case of *ex post* arbitrage, *ex ante* arbitrage will not eliminate price discrimination if either the probability of any consumer being *L* is high enough or if $\underline{v}$ and $\overline{v}$ are close enough to each other. Further, price discrimination in the presence of *ex ante* arbitrage depends on the possibility that the forward market will 'fail' due to a lack of *L* buyers. This mirrors the result for *ex post* arbitrage. In each case, price discrimination arises by *M* trading off the likelihood that the arbitrage market will operate efficiently and the potential gains from selling at a high price to an *H* consumer.

This said, the presence of *ex post* and *ex ante* arbitrage is likely to affect *M*'s pricing behavior in quite different ways. With *ex post* arbitrage, when price discriminating, *M* can only charge *H* consumers a price that does not exceed $\underline{v}$ by too much. An *H* consumer, when faced with this price, then has to weigh up the benefits of securing the product for certain in period 1 or risk waiting to see if they can purchase the product at a cheaper price in period 2.

In contrast, with *ex ante* arbitrage, the relevant forward trade occurs prior to the consumers facing *M*. As such, the best that *M* can do is to always charge a high price to *H* consumers. The trade-off for *M* is whether or not it should set a low price to the *L* consumers at all, knowing that this may lead to a fall in high-price sales as some *H* consumers may be holding relevant forward contracts. As a result, if price discrimination occurs under *ex ante* arbitrage, the prices set are identical to those under perfect price discrimination. In contrast, with price discrimination in the presence of *ex post* arbitrage, the *H* consumer price will be reduced to enhance the trade-off facing the *H* consumer.

In summary, when the monopolist offers discriminatory pricing under ex ante arbitrage, forward contracts 'turn' an *H* into an *L*; something that would not happen if you offered a single price. So the trade off is akin to the situation without price discrimination - either set a high price and sell to all *H* and no *L*s or price discriminate and sell at a high price to any *H* without a forward contract and a low price to everyone else. In comparison, the standard no price discrimination alternative is just an extreme version of this: set a high price only and sell to all *H*s but no *L*s or set a low price and sell to everyone at the low price.

5. Conclusions

We have demonstrated here that costless arbitrage does not necessarily constrain the practice of price discrimination. When consumers and the seller do not expect that the number of low valuation consumers will be large, the seller finds it optimal to charge low and high value consumers different prices. Even when consumers can hedge against such risks in forward markets, the monopolist will still use observationally perfect price discrimination if it suspects that the number of potential arbitrageurs is low. This suggests that, at the very least, the textbook condition that arbitrage be impossible or prohibitively costly can be amended with an emphasis on the mass of consumers likely to be of low valuation and to engage in arbitrage activities. Specifically, so long as there is a non-zero probability that there are *L*-types in the population, price discrimination is feasible.

To our knowledge, this is the first attempt to consider in detail this particular assumption underlying the feasibility of price discrimination – either first or third degree. While the model is an extreme one in that there is perfect observability of types and very strong re-sale market competition, we believe that the insights from this model may be useful in studies of price discrimination. In particular, strategies that limit *effective* (as opposed to actual) arbitrage may be usefully employed by sellers wishing to charge different prices to different customers. These include strategies that obscure the nature of discounts being offered and who they are being offered to, their time sensitivity (as in Amazon.com's Gold Box) and also the volume of purchases allowed at discounted prices. Each of these limit effective arbitrage.

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